

Quantum Eraser Using Spin-1/2 Particles

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Most of the experimental realizations of quantum eraser till now, use photons. A new setup to demonstrate quantum eraser is proposed, which uses spin-1/2 particles in a modified Stern-Gerlach setup, with a double slit. When the *which-way* information is erased, the result displays two interference patterns which are transverse shifted. Use of the classic Stern-Gerlach setup, and the unweaving of the washed out interference without any coincident counting, is what makes this proposal novel.

PACS numbers: 03.65.Ud ; 03.65.Ta

It is well known that particles and light both, are capable of exhibiting a dual nature. This is commonly referred to as wave-particle duality. What is not emphasized commonly, is the fact that these natures are mutually exclusive - for example, light can act either as a particle, or as a wave at a time. This has its foundation in Bohr's complementarity principle [1]. It can be best understood in the context of Young's double slit experiment with particles. Complementarity principle implies that in such an experiment, there is a fundamental incompatibility between the "*Welcher-Weg*", or which-way information and the observation of interference pattern. Thus any attempt to obtain information about which slit the particle went through, necessarily destroys the interference pattern. Replying to Einstein's famous thought experiment regarding a recoiling double-slit, Bohr had demonstrated that the uncertainty in the initial position of the double-slit is precisely enough to wash out the interference pattern.

However, it turns out that it was just fortuitous that the uncertainty principle seemed to wash out the interference pattern. It has been argued that one could have the which-way information without appreciably affecting the spatial part of the wave function of the particle [2]. This can be done by entanglement of the particle with a variable, playing the role of a which-way marker. So, uncertainty principle is not the fundamental reason for washing out of interference in a double-slit experiment - entanglement is.

The double-slit experiment, with entanglement can be understood in the following way. Let us now assume that the initial state of the particle was entangled with a certain degree of freedom so that the state can be written as:

$$|\psi(r)\rangle = \frac{1}{\sqrt{2}}[|\psi_1(r)\rangle|1\rangle + |\psi_2(r)\rangle|2\rangle], \quad (1)$$

where $|1\rangle$ and $|2\rangle$ are certain normalized and orthogonal states, and $|\psi_1\rangle$ and $|\psi_2\rangle$ represent possibilities of

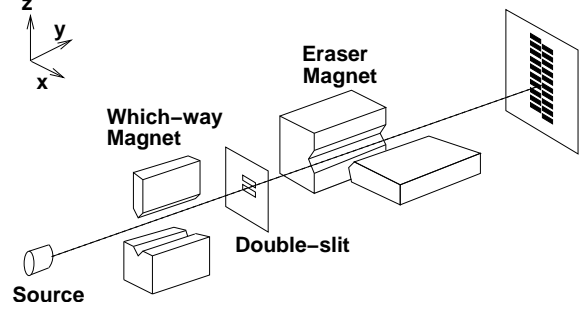


FIG. 1: Schematic diagram of proposed quantum eraser. Magnet 1 splits the beam into two so that they impinge on the double-slit. Magnet 2 splits the interfering beams by pulling apart the eigenstates of the x-component of the spin.

the particle going through one or the other slit. It is easy to see that when one calculates probability distribution of the particle on the screen $|\psi(r)|^2$, the cross-terms, $\psi_1^*(r)\psi_2(r)$ and $\psi_2^*(r)\psi_1(r)$, which are responsible for interference, are killed by the orthogonality of $|1\rangle$ and $|2\rangle$.

An interesting idea was put forward by Jaynes [3], and later independently by Scully and Drühl [2] saying that if the which-way information is stored in quantum detectors, it could also be erased by a suitable "reading out" of the detectors. In this situation, it should be possible to get back the interference. This came to be known as the *quantum eraser* [2, 3]. Scully, Englert and Walther proposed an experiment with Rydberg atoms, with micro-maser cavity detectors acting as which-way markers. They argued that if one were to perform a correlated measurement of the two detectors in such a way that the which-way information is lost, the interference pattern will be visible again [4].

Quantum eraser has been experimentally realized by various people using photons [5, 6, 7, 8, 9, 10, 11], mainly because it is easy to produce entangled photons via spontaneous parametric down conversion (SPDC). There have been some other proposals regarding NMR analogue of quantum eraser [12], neutral kaons [13] and cavity QED [14].

Here we propose an implementation of quantum eraser using a modified Stern-Gerlach setup with spin-1/2 par-

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ticles. As one will see later, the nice feature of this implementation is that it involves particles, instead of photons, and quantum erasing is vividly brought out by two sets of interference fringes on a real screen. No coincident counting of any kind is needed.

The setup consists of a Stern-Gerlach setup and a source of spin-1/2 particle (see Fig. 1). The particle travels along the positive y-axis, and the magnetic field is along the z-direction. After the Stern-Gerlach magnet, which we will call, the *which-way magnet*, there is a double-slit, kept such that the slits are parallel to the x-axis. Normally, in a double-split experiment, the position spread of the state of the particle (along the direction of the slit) when it reaches the slit, should be larger than distance between the two slits. Only then will both the slits “see” the particle at the same time. On the other hand, in the present setup, the initial position spread of the particle state has to be much smaller than the distance between the slits. The particle starts out in a spin state $\frac{1}{\sqrt{2}}(|S_z; +\rangle + |S_z; -\rangle)$, where $|S_z; \pm\rangle$ represent the eigenstates of \hat{S}_z , the z-component of the spin.

The magnetic field of the which-way magnet entangles the position spatial wave-function of the particle with the spin-states. Let us assume that the state of the particle, when it reaches the double slit, is

$$|\Psi_i\rangle = \frac{1}{\sqrt{2}}(|S_z; +\rangle|\phi_+\rangle + |S_z; -\rangle|\phi_-\rangle), \quad (2)$$

where $|\phi_\pm\rangle$ correspond to spatial wave functions centered at the upper and the lower slit, respectively. When particle crosses the slits, which are much narrower than the position spread of $|\phi_\pm\rangle$, the state which emerges on the other side of the slits consists of wave-packets which are localized in a much narrower region space. Consequently these packets spread much faster as the particle travels in time. Beyond a certain distance after the double-slit, the wave packets would have spread enough to overlap strongly with each other. Suppose that the state of the particle, at this time, is given by

$$|\Psi_f\rangle = \frac{1}{\sqrt{2}}(|S_z; +\rangle|\psi_+\rangle + |S_z; -\rangle|\psi_-\rangle), \quad (3)$$

where $|\psi_\pm\rangle$ represent very spread out wave packets, which strongly overlap with each other. This is the region in which interference is expected, if which-way information is not there. Although the wave-packets overlap with each other, each carries a which-way marker with it, in the form of the spin states $|S_z; \pm\rangle$. Thus, if this particle is made to fall on a screen, no interference will be seen. This can be verified by calculating the probability of finding the particle at a position along the z-axis, which should be

$$|\Psi_f(z)|^2 = \frac{1}{2}(|\psi_+(z)|^2 + |\psi_-(z)|^2). \quad (4)$$

Let us now introduce another Stern-Gerlach magnet, which we call the *eraser magnet*, with field along the x-axis. The effect of the eraser magnet will be to spatially

separate out the components of the state $|\Psi_f\rangle$ depending on spin eigenstates $|S_x; \pm\rangle$. In order to analyze what will happen in such a situation, let us write the state $|\Psi_f\rangle$ in terms of $|S_x; \pm\rangle$:

$$\begin{aligned} |\Psi_f\rangle &= \frac{|S_x; +\rangle + |S_x; -\rangle}{2}|\psi_+\rangle + \frac{|S_x; +\rangle - |S_x; -\rangle}{2}|\psi_-\rangle \\ &= |S_x; +\rangle \frac{|\psi_+\rangle + |\psi_-\rangle}{2} + |S_x; -\rangle \frac{|\psi_+\rangle - |\psi_-\rangle}{2}. \end{aligned} \quad (5)$$

The eraser magnet will cause the piece of the wave function correlated to $|S_x; +\rangle$ to shift along the positive x-axis, and that correlated to $|S_x; -\rangle$ to shift towards the negative x-axis. When the particle reaches the screen, the state acquires the form

$$|\Psi_e\rangle = |S_x; +\rangle \frac{|\psi_{1+}\rangle + |\psi_{1-}\rangle}{2} + |S_x; -\rangle \frac{|\psi_{2+}\rangle - |\psi_{2-}\rangle}{2}, \quad (6)$$

where subscript 1 indicates a wave-packet shifted towards the positive x-direction, and the subscript 2 indicates a wave-packet shifted towards the negative x-direction. The strength of the eraser magnetic field and the position of the screen are so adjusted that the wave-packets with opposite shifts along the x-axis, have negligible overlap, i.e., $\langle\psi_{1\pm}|\psi_{2\pm}\rangle = 0$.

Let us calculate the probability of finding the particle at a point (x, z) on the screen. Keeping in mind the orthogonality of $|S_x; \pm\rangle$, this is given by

$$\begin{aligned} |\Psi_e(x, z)|^2 &= \frac{1}{4}|\psi_{1+}(x, z)|^2 + \frac{1}{4}|\psi_{1-}(x, z)|^2 \\ &+ \frac{1}{4}\psi_{1+}^*(x, z)\psi_{1-}(x, z) + \frac{1}{4}\psi_{1-}^*(x, z)\psi_{1+}(x, z) \\ &+ \frac{1}{4}|\psi_{2+}(x, z)|^2 + \frac{1}{4}|\psi_{2-}(x, z)|^2 \\ &- \frac{1}{4}\psi_{2+}^*(x, z)\psi_{2-}(x, z) - \frac{1}{4}\psi_{2-}^*(x, z)\psi_{2+}(x, z) \end{aligned} \quad (7)$$

So, how is the situation different from what it was before? If there were no eraser magnet, $\psi_{1\pm}(x, z)$ would be the same as $\psi_{2\pm}(x, z)$, and the cross terms in (7) would cancel out. In this situation (7) would be identical to (4), which would mean, no interference. But with the eraser magnet on, the terms with subscripts 1 and 2 in (7) represent *two interference patterns* which are at different locations on the x-axis! This is quantum erasure because particles reaching the two different locations on the x-axis have spin-states $|S_x; +\rangle$ and $|S_x; -\rangle$ respectively, each of which possess no which-way information by themselves.

The two interference patterns appear identical, but because the cross-terms in one are with a negative sign, there would be a slight difference. It can be demonstrated that the difference would be that one interference pattern is vertically shifted with respect to the other, by one fringe. This makes the two interference patterns complementary, in the sense that the two combined result in no fringes, only continuously varying intensity. This is

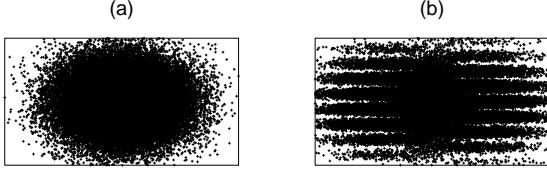


FIG. 2: Probability density distribution of particles hitting the screen (a) when the eraser magnet is switched off, and (b) when the eraser magnet is switched on.

of course expected, and is also seen in other implementations of quantum eraser.

If the analysis till now appears speculative, one can exemplify it by a rigorous calculation. Let us start from the stage at which the particle wave packets emerge from the double slit. We assume that the packets are Gaussian. The state of the particle can be written down as

$$\Psi(x, z) = Ae^{-\frac{x^2}{4\Omega^2}} \left(|S_z; +\rangle e^{-\frac{(z-z_0)^2}{4\sigma^2}} + |S_z; -\rangle e^{-\frac{(z+z_0)^2}{4\sigma^2}} \right) \quad (8)$$

where $A = \frac{1}{\sqrt{4\pi\sigma\Omega}}$. It represents two Gaussians centered at $z = \pm z_0$, where $2z_0$ is the distance between the two slits. The Gaussians centered at $z = \pm z_0$ are entangled with the spin states $|S_z; \pm\rangle$ respectively.

Suppose that the momentum in the y direction is such that the particle takes a time t to reach the screen. During this evolution, the packets would have spread. We let the state evolve under the influence of a free Hamiltonian $p_x^2/2m + p_z^2/2m$. The state of the particle, at a time t is given by

$$\Psi(x, z, t) = A_t e^{-\frac{x^2}{4(\Omega^2 - \frac{i\hbar}{m})}} \times \left(|S_z; +\rangle e^{-\frac{(z-z_0)^2}{4(\sigma^2 - \frac{i\hbar}{m})}} + |S_z; -\rangle e^{-\frac{(z+z_0)^2}{4(\sigma^2 - \frac{i\hbar}{m})}} \right), \quad (9)$$

where $A_t = \frac{(4\pi)^{-1/2}}{\sqrt{(\sigma - \frac{i\hbar}{m\sigma})(\Omega - \frac{i\hbar}{m\Omega})}}$. The probability density of the particle hitting the point (x, z) on the screen is given by

$$|\Psi(x, z, t)|^2 = |A_t|^2 e^{-\frac{x^2}{2(\Omega^2 + \frac{t^2\hbar^2}{m^2\sigma^2})}} \times \left(e^{-\frac{(z-z_0)^2}{2(\sigma^2 + \frac{t^2\hbar^2}{m^2\sigma^2})}} + e^{-\frac{(z+z_0)^2}{2(\sigma^2 + \frac{t^2\hbar^2}{m^2\sigma^2})}} \right) \quad (10)$$

This represents no interference of particles, which is expected because each particle carries with itself a which-way marker in the form of spin state $S_z; \pm\rangle$ (see Fig. 2(a)).

In order to see what happens when the eraser magnet is switched on, we write the state of the particle in terms

of the eigenstates of \hat{S}_x , $S_x; \pm\rangle$:

$$|\Psi_f\rangle = \frac{1}{\sqrt{2}} |S_x; +\rangle A e^{-\frac{x^2}{4\Omega^2}} \left(e^{-\frac{(z-z_0)^2}{4\sigma^2}} + e^{-\frac{(z+z_0)^2}{4\sigma^2}} \right) + \frac{1}{\sqrt{2}} |S_x; -\rangle A e^{-\frac{x^2}{4\Omega^2}} \left(e^{-\frac{(z-z_0)^2}{4\sigma^2}} - e^{-\frac{(z+z_0)^2}{4\sigma^2}} \right) \quad (11)$$

The particle starts out from the double-slit at time $t = 0$ and enters the region of the eraser magnet at $t = t_i$ and leaves it at time $t = t_i + t_e$. Then it travels and reaches the screen at a time t . Outside the region of the eraser magnet, the Hamiltonian governing the particle is given by $\hat{H}_f = \hat{p}_x^2/2m + \hat{p}_z^2/2m$. Within the region of the eraser magnet, the particle experiences an inhomogeneous magnetic field in the x -direction. The Hamiltonian in this region is given by $\hat{H}_e = \hat{p}_x^2/2m - \beta x \sigma_x + \hat{p}_z^2/2m$. The time evolution under the influence of this Hamiltonian can be worked out explicitly [15]. The state of the particle, when it reaches the screen is given by [15]

$$\Psi_e(x, z, t) = \frac{1}{\sqrt{2}} A_t |S_x; +\rangle e^{-\frac{(x - \frac{\beta}{2m} t_e^2)^2}{4(\Omega^2 - \frac{i\hbar}{m})} + \frac{i\beta t_e x}{\hbar}} \times \left(e^{-\frac{(z-z_0)^2}{4(\sigma^2 - \frac{i\hbar}{m})}} + e^{-\frac{(z+z_0)^2}{4(\sigma^2 - \frac{i\hbar}{m})}} \right) + \frac{1}{\sqrt{2}} A_t |S_x; -\rangle e^{-\frac{(x + \frac{\beta}{2m} t_e^2)^2}{4(\Omega^2 - \frac{i\hbar}{m})} - \frac{i\beta t_e x}{\hbar}} \times \left(e^{-\frac{(z-z_0)^2}{4(\sigma^2 - \frac{i\hbar}{m})}} - e^{-\frac{(z+z_0)^2}{4(\sigma^2 - \frac{i\hbar}{m})}} \right), \quad (12)$$

The above expression has a simple interpretation. The first term represents a Gaussian centered at $x = \frac{1}{2} \frac{\beta}{m} t_e^2$, which is just the distance traveled by a particle in time t_e with an acceleration β/m . The third term represents a Gaussian centered at $x = -\frac{1}{2} \frac{\beta}{m} t_e^2$, which is the distance traveled by a particle in time t_e with an acceleration $-\beta/m$. The terms $\exp(\pm \frac{i}{\hbar} \beta t_e x)$ indicate that momentum of the particle is $\pm \beta t_e$ which should be the momentum acquired by a particle after being accelerated for a time t_e with an acceleration $\pm \beta/m$. The probability density of the particle hitting the screen at a point (x, z) is given by

$$|\Psi_e(x, z, t)|^2 = \frac{1}{2} |A_t|^2 e^{-\frac{(x - \frac{\beta}{2m} t_e^2)^2}{2(\Omega^2 + \frac{t_e^2\hbar^2}{m^2\Omega^2})}} \times \{P_+(z) + P_-(z) + 2f(z)\} + \frac{1}{2} |A_t|^2 e^{-\frac{(x + \frac{\beta}{2m} t_e^2)^2}{2(\Omega^2 + \frac{t_e^2\hbar^2}{m^2\Omega^2})}} \times \{P_+(z) + P_-(z) - 2f(z)\} \quad (13)$$

where $P_{\pm}(z) = \exp\left[-\frac{(z \mp z_0)^2}{2(\sigma^2 + \frac{t_e^2\hbar^2}{m^2\sigma^2})}\right]$ and $f(z) = \exp\left[-\frac{(z^2 + z_0^2)}{2(\sigma^2 + \frac{t_e^2\hbar^2}{m^2\sigma^2})}\right] \cos\left[\frac{2zz_0\hbar/m\sigma^2}{\sigma^2 + \frac{t_e^2\hbar^2}{m^2\sigma^2}}\right]$. We plot this distribution and find that indeed the interference pattern,

which was lost because of the which-way information, now appears because the eraser magnet has erased the which-way information (see Fig. 2(b)). Notice that the z -position of the dark fringe of one pattern is the same as a bright fringe of the other, so that if they were not x -shifted, they would have cancelled out.

A few points need to be made at this stage. Without the eraser magnet, even though the which-way information is carried by the particle, one might wonder if one can actually extract it. The answer is that it can be done by putting a magnet in place of the eraser magnet, which has a magnetic field pointing along the z -axis, *but inhomogeneous along the x -axis*. For example $\vec{B} = \hat{k}B_0x$. This would cause the particles in states $|S_z; +\rangle$ and $|S_z; -\rangle$ to separate out along the x -axis.

One might also wonder if it is possible to perform this experiment in the “delayed” mode, meaning erasing the which-way information *after* the particle position has been registered. That is in principle possible, but might be a bit difficult to realize. All one needs is a special kind of screen between the double-slit and the eraser magnet, which registers the positions of the particles in the x - z plane, but still allows them to pass through, maybe with a reduced momentum - something like a cloud chamber. In such a situation, the first screen will show no interference pattern, because the which-way information exists. The second screen will show the two shifted interference patterns, because the which-way information has been erased.

It may be instructive to compare this proposal with other implementations of quantum eraser. In other implementations, one doesn’t get the interference directly and has to do a coincident counting of particles with certain states of the which-way detectors. Some people have this feeling, that the interference pattern is actually lost for good and one is only picking it out from the erased

pattern in an artificial way. In this respect, this method has the advantage that one can observe the interference appear right before one’s eyes as the eraser magnet is switched on. Another point is that this method allows one to demonstrate quantum erasure using massive particles, instead of photons.

Lastly, we would like to present our view on how one should understand the phenomena of quantum eraser. One might venture to ask, which path did the particle actually take. The eraser magnet in our setup, acts *after* the particle has crossed the slits. By finding the spin state of the particle $|+\rangle$ one might insist that the particle passed through only the upper slit. In our view, that is reading more meaning than what quantum mechanics provides. If you only look at quantum mechanical possibilities which have distinguishable which-way labels, those will not give you interference. If you only look at possibilities which have identical which-way labels you can get interference. Remember that even when the particle has crossed the slits, all possibilities do exist, before we have measured the particle positions *and* the spin state. So, it is not our act of reading the spin which dictates what the particle will do - rather, it is what we decide to look at finally, might *make it appear* as if the particle had taken one path or the other. What the particle *actually does* is not what the wave-function describes - it only describes the probability of outcomes. For a lively debate on this, see [16, 17].

In conclusion, we have proposed a new implementation of quantum eraser using spin-1/2 particles in a modified Stern-Gerlach setup. The which-way information can be erased simply by applying a magnetic field, and two complementary interference patterns appear on the screen. The method doesn’t require any fancy setup or entangled sources, except a Stern-Gerlach apparatus.

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